Module 2: Mathematical Foundations for Science

Introduction

Science uses mathematics as a tool to describe the behavior of the natural world.

Students sometimes believe that science courses require memorizing formulas. In reality, science requires understanding how physical quantities relate to one another.

Mathematics provides a precise language for expressing those relationships.

For this reason, students studying science must be comfortable using mathematical skills such as:

- algebra
- solving equations
- interpreting graphs
- unit conversions

In science courses, mathematics is not simply about calculation. It is about understanding how variables relate to each other.

The Role of Mathematics in Science

Scientific laws are often expressed in the form of equations.

For example, in physics the relationship between distance, velocity, and time is written as:

$v = d / t$

This equation tells us that velocity depends on how far an object travels and how long the motion takes.

Using equations allows scientists to make predictions about physical systems.

For example:

If we know the velocity of a moving object and the time it travels, we can calculate the distance traveled.

Algebraic Manipulation

Many scientific problems require solving equations for different variables.

Students must be comfortable rearranging equations.

For example:

$v = d / t$

If we want to solve for distance, we multiply both sides by time.

$d = vt$

If we want to solve for time:

$t = d / v$

Being able to rearrange equations is essential when solving science problems.

Worked Example

Suppose a car travels at a velocity of 25 meters per second for 8 seconds.

How far does the car travel?

The relationship between distance, velocity, and time is:

$d = vt$

Substitute values:

$d = 25 \times 8$

$d = 200$ meters

The car travels 200 meters.

Identifying Variables

In scientific equations, each symbol represents a physical quantity.

For example:

v = velocity
d = distance
t = time

Before solving a problem, students should identify which quantities are known and which must be calculated.

This step helps determine which equation should be used.

Example: Identifying Known and Unknown Quantities

A ball rolls down a ramp and reaches a velocity of 10 m/s after 4 seconds.

Find the acceleration.

Known:

velocity = 10 m/s
time = 4 s

Unknown:

acceleration

Relevant equation:

$v = at$

Solve for acceleration:

$a = v / t$

Substitute values:

$a = 10 / 4$

$a = 2.5\ m/s^2$

Graphs in Science

Graphs are frequently used in science to represent relationships between variables.

For example:

A distance vs. time graph shows how an object's position changes with time.

The slope of this graph represents velocity.

Understanding Graphs

Students should be able to interpret graphs by identifying:

- the variables on each axis
- the slope of the graph
- patterns in the data

Graph interpretation is an important skill in science.

Scientific Calculators

Scientific calculations often require the use of a scientific calculator.

Students should become familiar with their calculator's functions, including:

- exponents
- square roots
- parentheses
- scientific notation

Learning to use a calculator properly can reduce calculation errors.

Order of Operations

Mathematical expressions must be evaluated in the correct order.

The standard order is:

1. Parentheses
2. Exponents
3. Multiplication and division
4. Addition and subtraction

This order ensures that calculations produce consistent results.

Example

Evaluate the expression:

$3 + 4 \times 2^2$

Step 1
Calculate exponent.

$2^2 = 4$

Step 2
Multiply.

$4 \times 4 = 16$

Step 3
Add.

$3 + 16 = 19$

Final answer:

19

Units in Science

Units are essential in scientific calculations.

A numerical value without units does not provide meaningful information.

For example:

10 could represent:

- 10 meters
- 10 seconds
- 10 kilograms

Always include units in scientific calculations.

Example With Units

A car travels at 30 m/s for 5 seconds.

Distance:

$d = vt$

$d = 30 \times 5$

$d = 150$ meters

The unit of the answer is meters, because velocity was in meters per second and time was in seconds.

Estimation and Reasonableness

After solving a problem, students should ask whether the answer makes sense.

For example:

If a student calculates that a person runs 5000 meters in 2 seconds, the answer is clearly unrealistic.

Checking the reasonableness of answers helps detect errors.

Common Mathematical Mistakes

Students sometimes make errors that interfere with solving science problems.

Common mistakes include:

- forgetting units
- algebra errors
- incorrect order of operations
- incorrect equation rearrangement

Carefully writing each step helps avoid these errors.

Practice Problems

1. Rearrange the equation $v = d/t$ to solve for distance.
2. A car travels at 15 m/s for 6 seconds.
   Find the distance traveled.
3. Rearrange the equation $v = v_o + at$ to solve for acceleration.
4. Evaluate:

   $$5 + 3 \times 2^2$$

5. A bicycle travels 120 meters in 10 seconds.
   Find its velocity.

Challenge Problems

1. A car travels at 20 m/s for 12 seconds.
   How far does it travel?
2. Rearrange the equation $F = ma$ to solve for mass.
3. If a runner covers 200 meters in 25 seconds, what is the velocity?