

# Module 4: Dimensional Analysis and Unit Conversions

## Introduction

Scientific measurements always include both a number and a unit.

For example:

- 5 meters
- 10 seconds
- 3 kilograms

The number tells us the magnitude of the quantity, while the unit tells us what type of quantity is being measured.

When solving science problems, students often need to convert between different units. A systematic method known as dimensional analysis makes these conversions reliable and easy to follow.

Dimensional analysis is widely used in chemistry, physics, biology, and earth science. Developing skill with this method will help students solve many types of scientific problems.

### What Is Dimensional Analysis?

Dimensional analysis is a method that uses conversion factors to change one unit into another.

A conversion factor is a ratio that represents the relationship between two equivalent quantities.

For example:

1 meter = 100 centimeters

From this relationship we can form two conversion factors:

1 m / 100 cm

100 cm / 1 m

Both represent the same relationship.

Choosing the correct conversion factor allows unwanted units to cancel during calculations.

### The Principle of Unit Cancellation

The key idea behind dimensional analysis is unit cancellation.

Units behave like algebraic quantities.

If the same unit appears in both the numerator and denominator, it cancels.

Example:

cm / cm = 1

This allows us to eliminate unwanted units step by step until we obtain the desired unit.

#### Example 1

Converting Centimeters to Meters

Convert 250 cm to meters.

We know:

1 meter = 100 centimeters

Start with the given quantity:

250 cm

Multiply by a conversion factor that cancels centimeters:

$250 \text{ cm} \times (1 \text{ m} / 100 \text{ cm})$

Cancel the units:

cm cancels with cm

Now perform the calculation:

$250 \div 100 = 2.5$

Answer:

2.5 meters

### **Example 2**

Converting Kilometers to Meters

Convert 3.2 km to meters.

We know:

1 km = 1000 m

Start with the given value:

3.2 km

Multiply by the conversion factor:

$3.2 \text{ km} \times (1000 \text{ m} / 1 \text{ km})$

Cancel the units:

km cancels with km

Calculate:

$3.2 \times 1000 = 3200$

Answer:

3200 meters

### Multi-Step Conversions

Sometimes a problem requires more than one conversion.

Dimensional analysis allows these conversions to be performed systematically.

#### Example 3

Converting Meters to Millimeters

Convert 0.75 meters to millimeters.

We know:

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ cm} = 10 \text{ mm}$$

Step 1

Convert meters to centimeters.

$$0.75 \text{ m} \times (100 \text{ cm} / 1 \text{ m})$$

$$= 75 \text{ cm}$$

Step 2

Convert centimeters to millimeters.

$$75 \text{ cm} \times (10 \text{ mm} / 1 \text{ cm})$$

$$= 750 \text{ mm}$$

Final answer:

750 millimeters

#### Example 4

Converting Hours to Seconds

Convert 2 hours to seconds.

We know:

$$1 \text{ hour} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

Start with the given value:

2 hours

Step 1

Convert hours to minutes.

$$2 \text{ hr} \times (60 \text{ min} / 1 \text{ hr})$$

= 120 minutes

Step 2

Convert minutes to seconds.

$$120 \text{ min} \times (60 \text{ s} / 1 \text{ min})$$

= 7200 seconds

Final answer:

7200 seconds

## Dimensional Analysis in Science

Dimensional analysis is used frequently in scientific calculations.

Examples include:

- converting units in chemistry experiments
- converting time units in physics problems
- converting concentrations in biology
- converting distances in earth science

Because dimensional analysis follows a clear logical structure, it helps prevent calculation errors.

### Example From Physics

Suppose a car travels at 20 meters per second.

Convert this velocity to kilometers per hour.

We know:

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ hour} = 3600 \text{ seconds}$$

Start with:

$$20 \text{ m/s}$$

Convert meters to kilometers:

$$20 \text{ m/s} \times (1 \text{ km} / 1000 \text{ m})$$

$$= 0.02 \text{ km/s}$$

Convert seconds to hours:

$$0.02 \text{ km/s} \times (3600 \text{ s} / 1 \text{ hr})$$

$$= 72 \text{ km/hr}$$

Answer:

72 kilometers per hour

### Checking Units

One advantage of dimensional analysis is that it helps verify whether an equation makes sense.

For example:

velocity = distance / time

Units:

meters / seconds

This matches the units of velocity.

If the units do not match the expected physical quantity, the equation is probably incorrect.

### Common Mistakes in Unit Conversions

Students often make mistakes when converting units.

Common errors include:

- using the wrong conversion factor
- forgetting to cancel units
- skipping steps in multi-step conversions

Carefully writing each step helps avoid these errors.

### Strategy for Solving Conversion Problems

Students should follow these steps when using dimensional analysis:

1. Write down the given quantity with its units.
2. Identify the unit you want to obtain.
3. Select a conversion factor that cancels the unwanted unit.
4. Multiply and simplify the units.
5. Perform the calculation.

This systematic approach works for nearly all unit conversion problems.

### Practice Problems

1. Convert 500 cm to meters.
2. Convert 4 km to meters.
3. Convert 750 g to kilograms.
4. Convert 90 minutes to hours.

5. Convert 3 hours to seconds.

### **Multi-Step Practice Problems**

1. Convert 2.5 meters to millimeters.
2. Convert 120 minutes to seconds.
3. Convert 5 kilometers to centimeters.
4. Convert 0.5 kilograms to grams.

### **Challenge Problems**

1. Convert 15 m/s to km/hr.
2. Convert 72 km/hr to m/s.
3. Convert 1.5 hours to seconds.