Activity P41: Waves on a String
(Power Amplifier)

What Do You Think?

The purpose of this activity is to investigate standing waves in a string. What is the relationship between the tension in the string and the number of segments in the standing wave? What is the relationship between the frequency of oscillation of the string and the number of segments in the standing wave? How can you use these relationships to find the linear mass density of the string?

Take time to answer the ‘What Do You Think?’ question(s) in the Lab Report section.

Background

When a stretched string is plucked it will vibrate in its fundamental mode in a single segment with nodes on each end. If the string is driven at this fundamental frequency, a standing wave is formed. Standing waves also form if the string is driven at any integer multiple of the fundamental frequency. These higher frequencies are called the harmonics.

Each segment is equal to half a wavelength. In general for a given harmonic, the wavelength \( \lambda \) is

\[
\lambda = \frac{2L}{n}
\]

where \( L \) is the length of the stretch string and \( n \) is the number of segments in the string.

The linear mass density of the string can be directly measured by weighing a known length of the string. The density is the mass of the string per unit length.

\[
\mu = \frac{\text{mass}}{\text{length}}
\]

The linear mass density of the string can also be found by studying the relationship between the tension, frequency, length of the string, and the number of segments in the standing wave. To derive this relationship, the velocity of the wave is expressed in two ways.

The velocity of any wave is given by \( v = \lambda f \) where \( f \) is the frequency of the wave. For a stretched string:

\[
v = \frac{2Lf}{n}
\]
The velocity of a wave traveling in a string is also dependent on the tension, \( T \), in the string and the linear mass density, \( \mu \), of the string:

\[
v = \sqrt{\frac{T}{\mu}}
\]

Setting these two expressions for the velocity equal to each other and solving for tension gives:

\[
T = \left(4L^2 f^2 \mu \right) \left(\frac{1}{n^2}\right)
\]

If the tension is varied while the length and frequency are held constant, a plot of tension, \( T \), vs. \((1/n^2)\) will give a straight line which will have a slope equal to \(4L^2 f^2 \mu\). The slope of this line can be used to calculate the linear mass density of the string.

The equation for the tension can also be solved for the frequency:

\[
f = \sqrt{\frac{T}{4L^2 \mu}} n
\]

If the frequency is varied while the tension and length are held constant, a plot of frequency, \( f \), vs. number of segments, \( n \), will give a straight line. The slope of this line can also be used to calculate the linear mass density of the string.

**Overview**

- First, determine the linear mass density of the string directly (**Pre-Lab**).
- Next, determine the linear mass density of the string using the relationship of tension, length, number of segments, and frequency for standing waves on the string.

In Part A use different hanging masses to change the tension of a string but keep the length and frequency constant. Plot a graph of tension vs. \(1/n^2\) to determine the linear mass density of the string.

In Part B use the wave driver to vary the frequency but keep the length and tension constant. Use DataStudio or ScienceWorkshop to control the frequency of the wave driver. Plot a graph of frequency vs. \( n \) to determine the linear mass density of the string.

Compare the values of linear mass density for all three methods.

**SAFETY REMINDER**

- Follow all safety instructions.
2. Calculate the linear mass density by dividing the mass by the length \((\mu = \text{mass/length})\): Record this value in the Lab Report section.
For You To Do

In Part A of this activity, use different hanging masses to change the tension in the string but keep the length and frequency constant. Use DataStudio or ScienceWorkshop to keep the wave driver frequency constant.

### Part A: Change Tension – Keep Length and Frequency Constant

**PART IA: Computer Setup**

1. Connect the ScienceWorkshop interface to the computer, turn on the interface, and turn on the computer.

2. Connect the Power Amplifier DIN plug into Analog Channel A of the interface.

3. Open the file titled as shown:

<table>
<thead>
<tr>
<th>DataStudio</th>
<th>ScienceWorkshop (Mac)</th>
<th>ScienceWorkshop (Win)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P41 Waves.DS</td>
<td>P31 Waves on a String</td>
<td>P31_WAVE.SWS</td>
</tr>
</tbody>
</table>

- The DataStudio document has Table and Graph displays of Tension and $1/n^2$ and Frequency and Segments. It also has a Workbook display. Read the instructions in the Workbook.
- The ScienceWorkshop document opens with a Signal Generator window.
- The Signal Generator is set to Auto. The Signal Generator output will begin when you start data recording and will stop when you end data recording.

**PART IIA: Sensor Calibration and Equipment Setup**

1. To avoid overloading the equipment, do not turn on the power switch of the Power Amplifier until the equipment setup is complete.

2. Set up the equipment. Tie one end of a 2-m long piece of string to a vertical support rod that is clamped to one end of a table. Pass the other end of the string over a pulley that is mounted on a rod that is clamped to
the other end of the table. Attach about 500 g to the end of the string.
3. Place the wave driver under the string near the vertical support rod. Insert the string in the slot on the top of the driver plug of the wave driver so the wave driver can cause the string to vibrate up and down. Use patch cords to connect the wave driver into the output jacks of the Power Amplifier.

4. Use the meter stick to measure the length of the section of the string, L, that will be vibrating (the part between the driver plug of the wave driver and the top of the pulley). Record this length in the Lab Report section, Table 1.

**PART IIIA: Data Recording – Change Tension**

1. Turn on the power switch on the back panel of the Power Amplifier.

2. Put enough mass on the mass hanger to make the string vibrate in its fundamental mode (one antinode in the center) at a frequency of 60 Hz. Adjust the amount of mass until the nodes at each end are “clean” (not vibrating). Record the initial mass in the Lab Report section, Table 1. (Be sure to include the mass of the hanger.)

3. Now change the amount of mass on the mass hanger until the string vibrates in each of the higher harmonics (for 2 segments through 8 segments) and record these masses in Table 1 section. **Hint:** Decrease the mass to increase the number of segments.

**Analyzing the Data: Change Tension**

1. Calculate the tension for each different mass used (tension = mass in kilograms x ‘g’ where g = 9.8 newtons per kilogram).

2. Plot a graph of Tension vs. 1/n^2.
   - **In DataStudio,** enter the values of Tension in the Table of Tension vs. 1/n^2. (Hint: The default values of 1/n^2 are already in one column of the Table. Enter a value for Tension and press <tab> or <return> to move to the next cell.)
   - **Note:** The data are automatically plotted in the Graph display of Tension vs. 1/n^2 as the values are entered in the Table.

   **In ScienceWorkshop,** do the following:
   - **Clear the ‘Notes’ window.**
   - **In the ‘Notes’ window,** enter the values of 1/n^2 and Tension as ordered ‘X’-‘Y’ pairs. Use the following format to type in the values:
     
     1/n^2 value <tab> Tension value <return>

     - When the ordered pairs are entered in the ‘Notes’ window, use ‘Select All’ from the ‘Edit’ menu to highlight the data. Select ‘Copy’ from the ‘Edit’ menu to copy the data.
• Click the Experiment Setup window to make it active. Select ‘Paste’ from the ‘Edit’ menu to paste the data pairs into the Data list. **Result:** The ‘Enter Data Cache Information’ window opens.

• Enter a ‘Long Name’ (e.g. Tension vs. 1/n^2), a ‘Short Name’ (e.g. Tension), and ‘Units’ (e.g. N). Click ‘OK’ to return to the Experiment Setup window. **Result:** The ‘Short Name’ appears in the Data list.

• Select ‘New Graph’ from the ‘Display’ menu. **Result:** A Graph display of Current vs. Time automatically opens.

• Use the ‘Input’ menu in the Graph display to select ‘Data Cache, Tension vs. 1/n^2’. **Result:** The Graph display will show ‘Tension vs. 1/n^2’ on the vertical axis. Note: The horizontal axis shows the correct values for 1/n^2 but the label remains ‘Time (s)’.

3. Determine the slope of the best-fit line on the Tension vs. 1/n^2 graph.

• In DataStudio click the ‘Fit’ menu and select ‘Linear’. **Result:** The slope appears in the Legend box. Note: To see the mathematical formula and its parameters, double-click the Legend box in the Graph.

• In ScienceWorkshop, click the ‘Statistics’ button to open the statistics area and use the ‘Statistics Menu’ to select ‘Curve Fit, Linear Fit’. **Result:** The coefficient ‘a2’ is the slope.

4. Using the slope, length, and frequency, calculate the linear mass density of the string. (Hint: The slope is equal to 4L^2 f^2 μ. Solve for the linear density.) Record the value in the Lab Report section, Table 1.

5. Calculate the percent difference between this value and the directly measured value and record it in the Lab Report, Table 3.
Part B: Change Frequency – Keep Tension and Length Constant

PART IB: Computer Setup

• Use the same setup as in the first part of this procedure.

PART IIB: Sensor Calibration and Equipment Setup

• Put 500 g on the mass hanger. Calculate and record this Tension in Table 2.

PART IIIB: Data Recording – Change Frequency

1. Vary the output frequency of the Signal Generator until the string vibrates in one segment (the fundamental frequency).
   • In DataStudio, use the ‘right-left’ arrows to change the frequency increment. Use the ‘minus-plus’ buttons to adjust the frequency.
   • In ScienceWorkshop, use the ‘up/down’ arrows to adjust the frequency.
   • In ScienceWorkshop, adjust the amount of frequency change for each ‘up/down’ click with the following keys:

<table>
<thead>
<tr>
<th>Key (with mouse click)</th>
<th>Frequency change</th>
<th>Key (with mouse click)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macintosh®</td>
<td>Windows™</td>
<td></td>
</tr>
<tr>
<td>Shift key</td>
<td>100 Hz</td>
<td>Shift key</td>
</tr>
<tr>
<td>&quot;No&quot; key</td>
<td>10 Hz</td>
<td>&quot;No&quot; key</td>
</tr>
<tr>
<td>Control key</td>
<td>1 Hz</td>
<td>Ctrl (control) key</td>
</tr>
<tr>
<td>Option key</td>
<td>0.1 Hz</td>
<td>Alt key</td>
</tr>
<tr>
<td>Command key</td>
<td>0.01 Hz</td>
<td>Ctrl + Alt keys</td>
</tr>
</tbody>
</table>

2. Find the frequencies required for the higher harmonics (n = 2 through 7) and record these in the Lab Report section, Table 2.
Analyzing the Data: Change Frequency – Constant Tension and Length

3. Plot a graph of Frequency vs. Segments.
   • In *DataStudio*, enter the values of Frequency in the Table of Frequency vs. Segments. (Hint: The default values of ‘n’ are already in one column of the Table. Enter a value for Frequency and press <tab> or <return> to move to the next cell.)
   • Note: The data are automatically plotted in the Graph display of Frequency vs. Segments as the values are entered in the Table.

   In *ScienceWorkshop*, do the following:
   • Clear the ‘Notes’ window.
   • In the ‘Notes’ window, enter the values of ‘n’ and Frequency as ordered ‘X’-’Y’ pairs. Use the following format to type in the values:
     \[ \text{n value} \quad \text{<tab>} \quad \text{Frequency value} \quad \text{<return>} \]
   • When the ordered pairs are entered in the ‘Notes’ window, use ‘Select All’ from the ‘Edit’ menu to highlight the data. Select ‘Copy’ from the ‘Edit’ menu to copy the data.
   • Click the Experiment Setup window to make it active. Select ‘Paste’ from the ‘Edit’ menu to paste the data pairs into the Data list. **Result:** The ‘Enter Data Cache Information’ window opens.
   • Enter a ‘Long Name’ (e.g. Frequency vs. n), a ‘Short Name’ (e.g. Frequency), and ‘Units’ (e.g. Hz). Click ‘OK’ to return to the Experiment Setup window. **Result:** The new ‘Short Name’ appears in the Data list.
   • Use the ‘Input’ menu in the Graph display to select ‘Data Cache, Frequency vs. n’. **Result:** The Graph display will show ‘Frequency vs. n’ on the vertical axis. Note: The horizontal axis shows the correct values for n but the label remains ‘Time (s)’.

4. Determine the slope of the best-fit line on the Frequency vs. n graph.
   • In *DataStudio* click the ‘Fit’ menu and select ‘Linear’. **Result:** The slope appears in the Legend box.
   • In *ScienceWorkshop*, click the ‘Statistics’ button to open the statistics area and use the ‘Statistics Menu’ to select ‘Curve Fit, Linear Fit’. **Result:** The coefficient ‘a2’ is the slope.

5. Using the slope, length, and frequency, calculate the linear mass density of the string.
   (Hint: The slope is equal to \( \frac{T}{4L^2 \mu} \). Solve for the linear density.) Record the value in the Lab Report section, Table 1.
6. Calculate the percent difference between this value and the directly measured value and record it in the Lab Report, Table 3.

**Record your results in the Lab Report section.**
Lab Report – Activity P41: Waves on a String

What Do You Think?

The purpose of this activity is to investigate standing waves in a string. What is the relationship between the tension in the string and the number of segments in the standing wave? What is the relationship between the frequency of oscillation of the string and the number of segments in the standing wave? How can you use these relationships to find the linear mass density of the string?

Data

Table 1: Change Tension – Constant Frequency and Length

<table>
<thead>
<tr>
<th>Segments, n</th>
<th>Mass (kg)</th>
<th>Tension, T (N)</th>
<th>1/n²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
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<td>3</td>
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<td>5</td>
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<td>7</td>
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<tr>
<td>8</td>
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<td></td>
</tr>
</tbody>
</table>

Linear mass density = __________ kg/m

Table 2: Vary Frequency

<table>
<thead>
<tr>
<th>Tension</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>__</td>
<td>__</td>
</tr>
</tbody>
</table>

Linear mass density = __________ kg/m

Table 3: Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Linear mass density</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension vs. 1/n²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Questions

1. As the tension is increased, does the number of segments increase or decrease when the frequency is kept constant?

2. As the frequency is increased, does the number of segments increase or decrease when the tension is kept constant?

3. As the tension is increased, does the speed of the wave increase, decrease, or stay the same when the frequency is kept constant?

4. As the frequency is increased, does the speed of the wave increase, decrease, or stay the same when the tension is kept constant?

5. Suppose that String #1 is twice as dense as String #2, but both have the same tension and the same length. If each of the strings is vibrating in the fundamental mode, which string will have the higher frequency?