PHY 1200 General Physics 2/PHY 1400 Advanced General Physics 2

You are responsible for remembering and being able to use EVERYTHING from PHY 1100/1300. PHY 1200/1400 is impossible to understand without the knowledge from PHY 1100/1300. If you do not remember PHY 1100/1300, you must review it immediately.

We will be doing even more calculations than in PHY 1100/1300. Have your scientific calculator ready!

Many topics in PHY 1200/1400 cover things that are invisible or abstract. There are many concepts in Physics that are defined Mathematically and cannot be properly interpreted in words. Exercise your imagination!

Part 1: Waves

What is a wave?

A wave is an OSCILLATION (vibration) that TRAVELS forward and carries ENERGY with it. A wave travels and is therefore different from, but is still mathematically related to an oscillator (which vibrate in place).

In a TRANSVERSE wave, the oscillation direction is PERPENDICULAR to the travel direction. Vibrating strings, water waves, light waves are common transverse waves.

In a LONGITUDINAL wave, the oscillation direction is PARALLEL to the travel direction. Sound waves are the most common longitudinal waves.

The constant wave traveling (or propagation) speed is abbreviated "v" and the unit is meters per second – just like any other velocity. The wave FREQUENCY is the number of vibrations per second. The symbol is "ν", the Greek lower case letter "nu," and the unit is cycles per second, called Hertz – Hz. (It's OK if you want to use "f" for frequency. A lot of people do, because "ν" looks too much like "v" for velocity.) Since a wave travels and vibrates at the same time, the WAVELENGTH is the distance the wave travels during one oscillation. The symbol is "λ", the Greek lower case letter "lambda," measured in meters. Note: we will be using many Greek letter abbreviations in this class, because we've run out of Roman letters.

Since these three things refer to one wave, the traveling speed, the frequency and the wavelength must be related to each other. The WAVE EQUATION for traveling speed is:

$$v = f\lambda \qquad \text{or} \qquad v = \lambda\,\nu$$

Sometimes, we measure the wave period, abbreviated "T", the time required for one complete oscillation – the opposite of frequency:

$$T = \frac{1}{f} \qquad \text{or} \qquad f = \frac{1}{T}$$

Therefore the wave equation can be rewritten:

$$v = \frac{\lambda}{T}$$

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wave functions

(PHY 1400 only)

However, for the oscillation, the wave "particle" POSITION (or DISPLACEMENT) FUNCTION (the equation of a wave) is:

$$y(x,t) = A\cos(kx - \omega t)$$

This function can be written as a sine function if desired, because sine and cosine are the same, except shifted in phase by $\frac{\pi}{2}$ radians (90°). Notice: this is a three-dimensional function – the oscillation position depends on both travel position and time.

Since velocity is the (partial) derivative of position over time, the oscillation VELOCITY FUNCTION is:

$$v(x,t) = \frac{\partial\, y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

Notice: the traveling speed is very different from the oscillation velocity.

Since acceleration is the (partial) derivative of velocity over time, the oscillation ACCELERATION FUNCTION is:

$$a(x,t) = \frac{\partial\ v(x,t)}{\partial t} = -\,\omega^2 A \cos(kx - \omega t) = -\,\omega^2\ y(x,t)$$

Since the wave number is $k = \dfrac{2\pi}{\lambda}$ and angular frequency is $\omega = 2\pi f$, we can rewrite:

displacement $\quad y(x,t) = A \cos\left(\dfrac{2\pi x}{\lambda} - 2\pi f t\right)$

velocity $\quad v(x,t) = \dfrac{\partial\ y(x,t)}{\partial t} = 2\pi f A \sin\left(\dfrac{2\pi x}{\lambda} - 2\pi f t\right)$

acceleration $\quad a(x,t) = \dfrac{\partial\ v(x,t)}{\partial t} = -\,4\pi^2 f^2 A \cos\left(\dfrac{2\pi x}{\lambda} - 2\pi f t\right)$

Note: the traveling wave equation above is an extremely simplified version of the equation.

Since $y(x,t) = A \cos(kx - \omega t)$

therefore $\quad \dfrac{\partial^2\ y(x,t)}{\partial x^2} = -\,k^2\ y(x,t) \quad$ and $\quad \dfrac{\partial^2\ y(x,t)}{\partial t^2} = -\,\omega^2\ y(x,t)$

Since also $\quad k = \dfrac{2\pi}{\lambda} \quad$ and $\quad \omega = 2\pi f$

therefore $\quad v = f\lambda \equiv \dfrac{\omega}{2\pi}\dfrac{2\pi}{k} = \dfrac{\omega}{k} \quad$ or $\quad v^2 = \dfrac{\omega^2}{k^2}$

$$\text{and} \quad v^2\ \frac{\partial^2\ y(x,t)}{\partial x^2} = \frac{\partial^2\ y(x,t)}{\partial t^2}$$

This is the wave equation as a second-order partial differential equation. (Since Kingsborough is a two-year college, PDE are not seriously discussed here. The highest level Kingsborough Math class is ordinary differential equations – ODE.) It is an important equation in many advanced Physics topics – there are many types of waves, as well as things that are not waves, but have wave-like periodic behavior.

string waves

String waves are a vibration on a string stretched under tension. They are transverse waves – vibration direction is perpendicular to traveling direction. String waves are most famous for musical instruments like guitars, violins and pianos.

The speed of a wave on a string is (without proof):

$$v_{string} = \sqrt{\frac{F_T}{\mu}} \equiv \sqrt{\frac{F_T}{{}^{m}/_{l}}}$$

where "μ", the Greek lower case letter "mu", is the linear mass density (mass per length). Notice: the higher the tension, the higher the wave speed and frequency (and vice versa), but the higher the linear density, the lower the wave speed and frequency (and vice versa).

sound waves

Sound waves are a mechanical pressure oscillation in matter. They are longitudinal waves – vibration direction is parallel to traveling direction. Be careful, sound waves in air are invisible – we hear sound, we don't see sound. You need to use your imagination. Note, we almost always sketch sound waves as transverse waves, even though they're not, because transverse waves are easier to draw.

The speed of sound in air depends on many factors – temperature being the most important:

$$v_{sound} \approx 331\ m/s + 0.60\ T\,(^\circ C)$$

$$v_{20\,^\circ C} \approx 343\ m/s$$

Sound intensity is measured by sound wave power.

$$I = \frac{P}{A}$$

The sound intensity level or decibel level (abbreviated "β", the Greek lower case letter "beta") – the loudness of a sound – is related to the sound intensity.

$$\beta = -10\log\frac{I}{I_0}$$

where the threshold intensity, $I_0 = 10^{-14}\ W/m^2$, is the lowest intensity that a human being can hear.

Note: since sound intensity level is calculated using a common logarithm, a 3 dB change is a doubling or halving of sound loudness.

Doppler effect

The frequency of a wave that an observer detects is the same that a source emits only if the observer and source are relative rest (not moving in relation to each other). If the observer and source are moving closer, the wavelength is compressed and the frequency is observed to be higher than the source. If the observer and source are moving further apart, the wavelength is stretched and the frequency is observed to be lower than the source. For sound waves:

$$f_{observer} = f_{source}\left(\frac{v_{sound} \pm V_{observer}}{v_{sound} \mp V_{source}}\right)$$

Note: there are alternate versions of this equation. Use the one you are most comfortable with.

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wave interference

It is possible to have two (or more) waves in the same place and at the same time. A common example is the shake one end of a string, fixed on the ends. The wave will travel down the string, bounce off the far end, and travel back. If the vibration on the original end continues, there are two waves with the same frequency on one string, traveling in opposite directions.

The two waves will INTERFERE each other. If the two waves are perfectly OUT OF PHASE (by 180° or π radians), they will cancel each other out. This is called perfect DESTRUCTIVE INTERFERENCE. The wave displacement can add to zero, and the wave energy dissipates as heat. This is how “noise cancelling headphones” work. They have an external microphone to pick up outside noise, which is analyzed by a built-in computer to add out-of-phase sound to the speaker and cancel out the noise, leaving only your music.

$$\cos\theta + \cos(\theta + \pi) = 0$$

If the two waves are perfectly IN PHASE, they will reinforce each other. This is called perfect CONSTRUCTIVE INTERFERENCE. The wave displacement and energy adds to a total.

$$\cos\theta + \cos\theta = 2\cos\theta$$

If the wave is continuously reinforced, the energy can add up to a large enough amplitude that the string can snap. (Look up the Tacoma Narrows Bridge. It's on YouTube.)

If the two waves are "in between" phases, they will add in a chaotic manner, which we will not worry about. The SUPERPOSITION (adding) of wave functions can be very complex, and is taught in Math class. (Look up "Fourier analysis".)

If two interfering waves have slightly different frequencies, there will be a BEAT FREQUENCY that gets louder and softer at the difference of frequencies.

$$f_{beat} = f_{higher} - f_{lower}$$

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wave harmonics

One very useful application of constructive interference is musical instruments. Guitar, violin, piano, etc. strings are tuned so that RESONANCE or HARMONIC patterns of specific frequencies vibrate on them – the musical notes.

Since the string is fixed at both ends, the wave cannot oscillate at the ends, producing nodes. Only certain frequencies and wavelengths will exactly fit the string, producing a STANDING WAVE, that looks like it's not traveling, but oscillating in place.

The resonant frequencies and wavelengths of each harmonic are:

$$f_n = n\frac{v}{2L} \qquad \text{and} \qquad \lambda_n = \frac{2L}{n} \qquad \text{where n = 1, 2, 3, …}$$

The coefficient (2) indicates that one-half of a wave is the largest pattern that can fit a string. The fundamental frequency (or first harmonic, n = 1) is the longest wave that fits – with one "loop". The higher numbered harmonics have higher frequencies and shorter wavelengths that are simple multiples of the fundamental. Theoretically, there are an infinite number of harmonics, but in reality a string can only bend and stretch so much before it breaks. Different mixes of the higher harmonics produce the different sounds of different musical instruments when they play the same notes.

Wind instruments, like flutes, trumpets and organs also produce resonance, but they are sound wave resonances in air. For an open pipe (open at both ends), displacement antinodes form at each end. Therefore, open pipes are symmetric like strings and compute the same way.

$$f_n = n\frac{v}{2L} \qquad \text{and} \qquad \lambda_n = \frac{2L}{n} \qquad \text{where n = 1, 2, 3, …}$$

However, a closed pipe (open at one end and closed at the other) has a displacement antinode at the open end and a displacement node at the closed end. Closed pipes are antisymmetric at compute differently.

$$f_n = n\frac{v}{4L} \qquad \text{and} \qquad \lambda_n = \frac{4L}{n} \qquad \text{where n = 1, 3, 5, … (odd numbers only)}$$

The coefficient (4) indicates that one-quarter of a wave (half a “loop”) is the largest that can fit a closed pipe.

Part 1: Waves Problems

Problem 1: The Apollo astronauts left three Lunar Ranging Retro-Reflector instruments on the Moon to reflect lasers fired from the Earth in order to measure the distance between the Earth and the Moon with centimeter accuracy. If the time for your laser beam to return is 2.55 676 284 s, how far is the reflector from your observatory? The constant speed of light is c = 299,792,458 m/s.

Solution: the time given is the total reflected time. The one-way time is:

$$t = \frac{2.55\ 676\ 284\ s}{2} = 1.27\ 838\ 142\ s$$

Since waves have constant speed:

$$x = vt$$
$$= 299{,}792{,}458\ m/s\ (1.27\ 838\ 142\ s)$$
$$x \approx 383{,}249{,}108\ m$$

Note: the continuing experiments show that the Moon is receding from the Earth at about 1 inch per year.

Problem: $y(x,t) = A\cos(kx - \omega t)$ What are the frequency, wavelength, traveling speed the oscillation speed?

Problem: An ambulance sounds its 600 Hz siren while driving toward a car at 15 m/s. If the car is driving in the opposite direction toward the ambulance at 10 m/s, what frequency does the car detect? Use speed of sound of 343 m/s.

Solution: the ambulance is the source and the car is the observer. Since the ambulance and car are moving closer, add on the top, subtract on the bottom.

$$f_{observer} = f_{source}\left(\frac{v_{sound} \pm V_{observer}}{v_{sound} \mp V_{source}}\right)$$

$$f_{observer} = 600\ Hz\left(\frac{343\ m/s + 15\ m/s}{343\ m/s - 10\ m/s}\right)$$

$$f_{observer} = 600\ Hz\left(\frac{358\ m/s}{333\ m/s}\right)$$

$$f_{observer} \approx 600\ Hz\,(1.075)$$

$$f_{observer} \approx 645\ Hz$$

Problem: A dolphin emits a sonar sound pulse of 60,000 Hertz at a stationary fish, while swimming toward the fish at 10 m/s. What frequency does the dolphin hear, after the sound pulse echoes back? The speed of sound in seawater is 1522 m/s.

Solution: this is a two part problem. First the sound pulse travels from the source dolphin to the observer fish. Then the distorted sound pulse reflects from the fish back to the dolphin – use the fish as source and the dolphin as observer.

dolphin to fish:

$$f_{observer} = f_{source}\left(\frac{v_{sound} \pm V_{observer}}{v_{sound} \mp V_{source}}\right)$$

$$f_{fish} = 60{,}000\ Hz\left(\frac{1522\ m/s}{1522\ m/s - 10\ m/s}\right)$$

$$f_{fish} = 60{,}000\ Hz\left(\frac{1522\ m/s}{1512\ m/s}\right)$$

$$f_{fish} \approx 60{,}000\ Hz\,(1.00661)$$

$$f_{fish} \approx 60{,}397\ Hz$$

fish to dolphin

$$f_{observer} = f_{source}\left(\frac{v_{sound} \pm V_{observer}}{v_{sound} \mp V_{source}}\right)$$

$$f_{dolphin} = 60{,}397\ Hz\left(\frac{1522\ m/s + 10\ m/s}{1522\ m/s}\right)$$

$$f_{dolphin} = 60{,}397\ Hz\left(\frac{1532\ m/s}{1522\ m/s}\right)$$

$$f_{dolphin} \approx 60{,}397\ Hz\,(1.00657)$$

$$f_{dolphin} \approx 60{,}794\ Hz$$

Part 2: Electric forces and fields

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electric charge

In PHY 1100/1300, mass in kilograms measures quantity of matter, but in electricity and magnetism, electric charge (abbreviated "q" or "Q") measures quantity of matter. Just as conservation of matter is true, so is conservation of charge.

There is a fundamental difference between mass and charge. There are two types of charge, called positive and negative charge; while there is only one type of mass, which can be called positive. Be careful: positive and negative charge does not equate to positive and negative number, although they do often compute that way. We say positive and negative charge to mean that they are somehow opposite.

The unit of charge is the Coulomb, C. Be careful, one Coulomb of charge is an enormous amount of charge. It's like using tons for mass. You can kill ten people with one Coulomb of charge. You will see milliCoulomb, $mC = 10^{-3}\ C$; microCoulomb, $\mu C = 10^{-6}\ C$; and nanoCoulomb, $nC = 10^{-9}\ C$ in realistic problems.

The elementary particles of charge are the microscopic protons and electrons that exist in atoms. They have equal ($e = 1.609 \times 10^{-19}\ C$), but opposite electric charge (protons are positive, electrons are negative). Since atoms normally have equal numbers of protons and electrons, normal objects are electrically neutral. If a real object has a net charge, there must have been a redistribution of the protons or electrons – normally electrons. Note: protons and electrons do not have equal mass: $m_p = 1.67 \times 10^{-27}\ kg$ and $m_e = 9.11 \times 10^{-31}\ kg$.

A conductor is any medium (material) that allows charges to move through it. An insulator is any medium (material) that blocks the movement of charges through it. Insulators are also called dielectrics, to indicate that electric fields can still extend through them. At normal conditions, metals are conductors, and nonmetals are insulators.

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electric forces

Coulomb's Law says that any two (or more) charges exert an electric force on each other:

$$F_E = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1\ q_2}{r^2}$$

or in vector notation:

$$\vec{F}_E = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1\ q_2}{r^2}\ \hat{r}$$

where "ε_0" (say "epsilon subscript zero", or "naught," if your professor was taught in Britain), is the permittivity of free space constant (the electric constant):

$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N \bullet m^2$$

$$k = \frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\ N \bullet m^2/C^2$$

Notice the Mathematical similarity with Newton's Law of Universal Gravitation:

$$F_g = G\frac{m_1\ m_2}{r^2} \qquad \text{with} \quad G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$$

This Mathematical similarity implies a deep Physical similarity, but Physicists don't know what it is yet. There are very important differences. Since there are both positive and negative charges, electric forces can be both attractive (pull closer) and repulsive (push apart), while gravitational forces are always attractive, because there is only positive mass. In addition, electric forces are generally 10^{30} times stronger than gravitational forces. However, since normal objects are electrically neutral, strong electric forces are uncommon.

All forces are vectors (they have magnitude and direction). The algebra version of Coulomb's Law is only computes for magnitude (be sure to IGNORE NEGATIVE SIGNS):

$$F_E = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1\ q_2}{r^2}$$

We give you rules for direction: OPPOSITE CHARGES ATTRACT; and LIKE CHARGES REPEL.

Make sure you remember how to add vectors.

electric fields

Electric forces are unusual. They are not contact forces. The charges don't touch, yet affect each other. Generally, objects must touch in PHY 1100/1300 for forces to exist. Telekinesis doesn't work for us. How does this work for charges?

Every charge fills the space around it with an invisible electric field (drawn as field lines to help us visualize what's going on), so that it can "reach out" and affect other charges. Coulomb's Law for the electric field of a point charge (in vector notation) is:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

Note: electric fields may be invisible to human beings, but there are creatures that can sense electric fields.

The unit of electric field is N/C (Newtons per Coulomb), or V/m (Volts per meter).

Electric fields are also vectors. The magnitude is computed (be sure to IGNORE NEGATIVE SIGNS):

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

The rules for direction are: AWAY FROM A POSITIVE CHARGE in all directions and TOWARD A NEGATIVE CHARGE in all directions.

When an "test" electric charge is in the electric field of a second charge, the test charge experiences an electric force.

$$\vec{F}_E = q\,\vec{E}$$

If the test charge is positive, the direction of the force and field are the same. If the test charge is negative, the direction of the force and field are the opposite.

Note: since electric fields are invisible and abstract, there is debate among Physicists about the "reality" of electric fields. Although the Math is valid, exactly what is the Physical meaning of particles and their fields is "not well understood." Physics explores many phenomena outside of normal human experience. We can determine the Mathematical rules they always obey, but that does not mean we can explain their

Physical meaning. Physics is not Math. Physics is the interpretation of how the Math explains reality.

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electric flux and Gauss' Law

(PHY 1400 only)

Coulomb's Law is only correct for point charges – charges of zero size, or charges so small compared to the distance to the charge that the charge size may be approximated as zero. If the charge size is large compared to charge distance, especially if you are inside the charge, you must use Gauss' Law:

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

The electric flux, abbreviated **Φ**, the Greek capital letter "Phi," which is the total electric field though a surface area, is proportional to the charge enclosed by the surface. Gauss' Law is often impractical to use, because real objects have complex shapes (and areas).

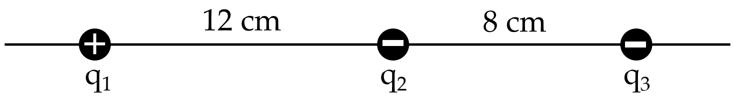
We will use simple shapes, such as spheres. There is an old Physics joke: "Assume the cow is a sphere." Remember: $A_{sphere} = 4\pi r^2$ and $V_{sphere} = \frac{4}{3}\pi r^3$

Note: there is a derivative version of Gauss' Law (which we won't use):

$$\nabla \bullet \vec{E} = 4\pi\rho$$

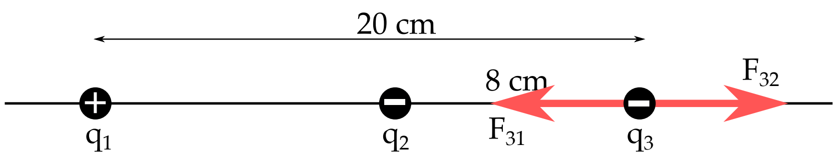
Part 2: Electric forces and fields Problems

Problem 1: Three charges $q_1 = +5\ \mu C$, $q_2 = -2\ \mu C$ and $q_3 = -8\ \mu C$ are fixed along a line as shown:



What is the net electric force on q_3?

Solution 1: I always say that the direction of a vector is more important than its magnitude. Therefore, apply the "opposite charges attract" and "like charges repel" rules to the diagram first:



Since there are three charges, there are two forces on q_3. Compute their magnitudes separately (remember to ignore the signs in the calculation – they were already used to determine direction), and then add them as vectors. I omit the units in the Math to save a little time.

$$F_{31} = \frac{1}{4\pi\varepsilon_0}\frac{q_3\ q_1}{r_{31}^2} \approx 9\times 10^9\ \frac{8\times 10^{-6}\left(5\times 10^{-6}\right)}{(0.20\ m)^2}$$

$$F_{31} \approx 9.0\ N$$

$$F_{32} = \frac{1}{4\pi\varepsilon_0}\frac{q_3\ q_2}{r_{32}^2} \approx 9\times 10^9\ \frac{8\times 10^{-6}\left(2\times 10^{-6}\right)}{(0.08\ m)^2}$$

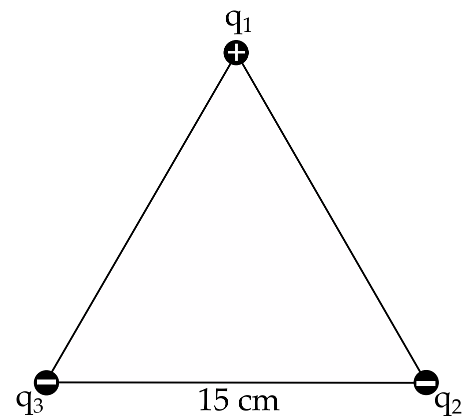
$$F_{32} \approx 22.5\ N$$

Notice: F_{31} points to the left – the negative direction, and F_{32} points to the right – the positive direction.

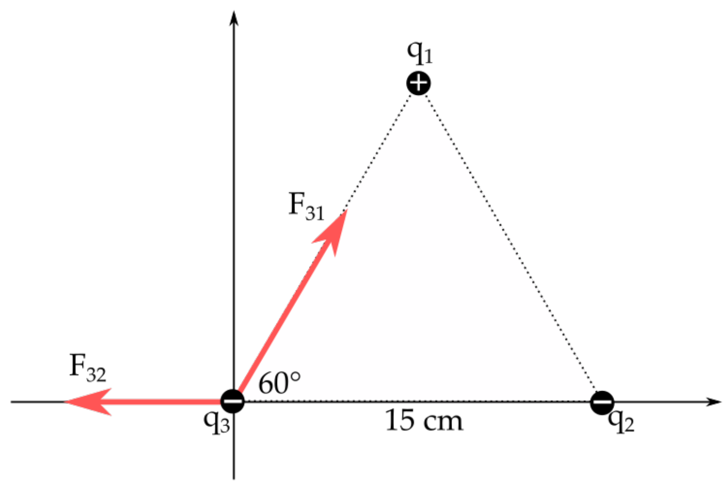
$$\vec{F}_{3_{net}} = \vec{F}_{31} + \vec{F}_{32} = (-9.0\ N) + (+22.5\ N)$$

$$\vec{F}_{3_{net}} = +13.5\ N$$

Problem 2: Three charges $q_1 = +5\mu C$, $q_2 = -2\ \mu C$ and $q_3 = -8\ \mu C$ are fixed in a 15 cm equilateral triangle as shown. What is the net electric force on q_3?



Solution 2: Confirm the vector directions first. I recommend putting the target charge at the origin of a x-y plane.



Now compute the force magnitudes. Remember to ignore the charge signs in magnitude calculations. I continue to omit the units in the Math.

$$F_{31} = \frac{1}{4\pi\varepsilon_0}\frac{q_3\ q_1}{r_{31}^2} \approx 9\times10^9\ \frac{8\times10^{-6}\left(5\times10^{-6}\right)}{(0.15)^2}$$

$$F_{31} \approx 16\ N$$

$$F_{32} = \frac{1}{4\pi\varepsilon_0}\frac{q_3\ q_2}{r_{32}^2} \approx 9\times10^9\ \frac{8\times10^{-6}\left(2\times10^{-6}\right)}{(0.15)^2}$$

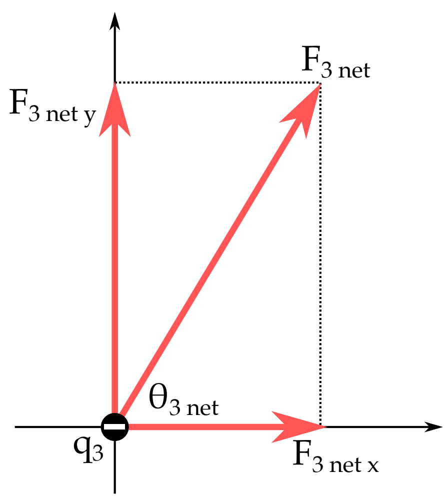
$$F_{32} \approx 6.4\ N$$

Finish by adding the two vectors. We use the "analytic" vector addition procedure from PHY 1100/1300.

When adding 2 or 3-D vectors, I recommend setting up a table to keep the vector components organized.

vector	x-axis (horizontal)	y-axis (vertical)
$\vec{F}_{31}$	$F_{31x} = F_{31}\cos\theta_{31}$ $= 16\cos 60^\circ$ $= 8\ N$	$F_{31y} = F_{31}\sin\theta_{31}$ $= 16\sin 60^\circ$ $\approx 13.86\ N$
$\vec{F}_{32}$	$F_{31x} = -6.4\ N$	$F_{31y} = 0$
$\vec{F}_{3_{net}}$	$F_{3\ net_x} = 1.6\ N$	$F_{3\ net_y} \approx 13.86\ N$

Finish by putting the components back together:



$$F_{3\ net} = \sqrt{F_{3\ net\ x}^2 + F_{3\ net\ y}^2}$$

$$F_{3\ net} = \sqrt{(1.6)^2 + (13.86)^2}$$

$$F_{3\ net} \approx 13.95\ N$$

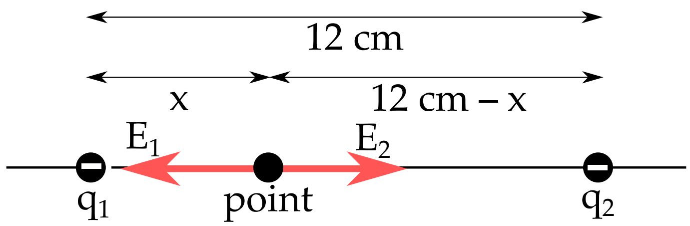
$$\theta_{3\ net} = \tan^{-1}\left(\frac{F_{3\ net\ y}}{F_{3\ net\ x}}\right)$$

$$\theta_{3\ net} = \tan^{-1}\left(\frac{13.86}{1.6}\right)$$

$$\theta_{3\ net} \approx 83.4^\circ$$

Problem 3: Two charges $q_1 = -3\ \mu C$ and $q_2 = -8\ \mu C$ are 12 cm apart. At what point between them is the net electric field equal to zero?

Solution: If two vectors are to add to zero, they must be equal in magnitude and opposite in direction. The opposite directions can be confirmed in a diagram; only the magnitudes need to be computed.



Since, we're setting up ratios, the Math is a little faster if you don't convert to standard units:

$$E_1 = E_2$$

$$\frac{1}{\cancel{4}\pi\varepsilon_0}\frac{q_1}{r_1^2} = \frac{1}{\cancel{4}\pi\varepsilon_0}\frac{q_2}{r_2^2}$$

$$\frac{3\ \mu C}{x^2} = \frac{8\ \mu C}{(12\ cm - x)^2}$$

$$\sqrt{\frac{3}{x^2}} = \sqrt{\frac{8}{(12-x)^2}}$$

$$\frac{1.732}{x} \approx \frac{2.828}{12-x}$$

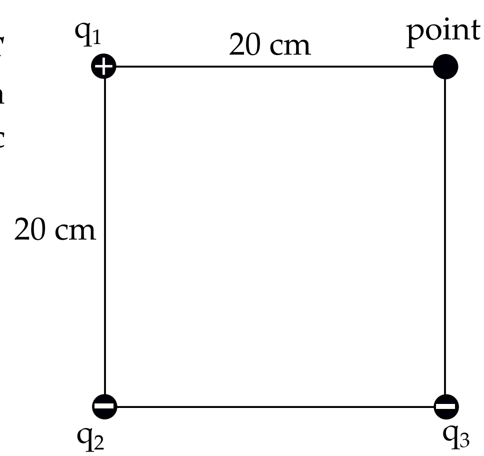
$$1.732\,(12-x) \approx 2.828x$$

$$20.78 - 1.732x \approx 2.828x$$

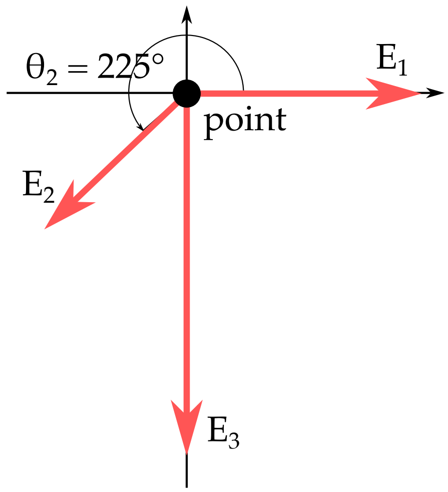
$$20.78 \approx 4.560x$$

$$\therefore \quad x \approx 4.6\ cm$$

Problem 4: Three charges $q_1 = +8\ \mu C$, $q_2 = -5\ \mu C$ and $q_3 = -12\ \mu C$ are fixed at three corners of a square, 20 cm on each side. What is the net electric field at a point on the other corner?



Solution: since electric field is a vector, you need a vector diagram with the point at the origin. Notice, q_2 is across the diagonal of the square to the point, so $r_2 \approx 28.3\ cm$.



Don't forget to ignore the charge signs in field magnitude calculations.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} \approx 9\times10^9\,\frac{8\times10^{-6}}{(0.20)^2}$$
$$E_1 \approx 1.8\times10^6\ N/C$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2} \approx 9\times10^9\,\frac{5\times10^{-6}}{(0.283)^2}$$
$$E_2 \approx 5.62\times10^5\ N/C$$

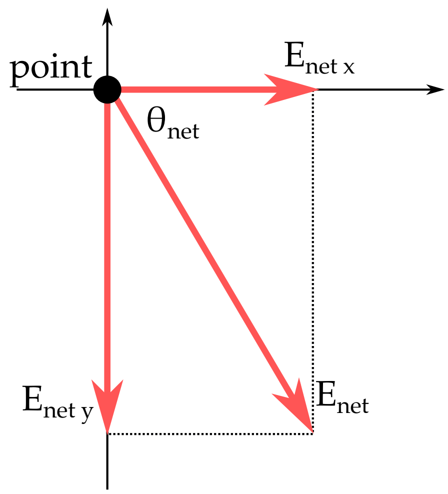
$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3^2} \approx 9\times10^9\,\frac{12\times10^{-6}}{(0.20)^2}$$
$$E_3 \approx 2.7\times10^6\ N/C$$

Keep the vector components organized with a table. Use the conventional angle if you want the directions to compute properly.

vector	x-axis (horizontal)	y-axis (vertical)
$\vec{E}_1$	$E_{1x} = 1.8\times10^6\ N/C$	$E_{1y} = 0$
$\vec{E}_2$	$E_{2x} = E_2\cos\theta_2$ $= 5.62\times10^5\cos 225^\circ$ $\approx -3.97\times10^5\ N/C$	$E_{2y} = E_2\sin\theta_2$ $= 5.62\times10^5\sin 225^\circ$ $= -3.97\times10^5\ N/C$
$\vec{E}_3$	$E_{3x} = 0$	$E_{3y} = -2.7\times10^6\ N/C$
$\vec{E}_{net}$	$E_{net_x} \approx 1.40\times10^6\ N/C$	$E_{net_y} \approx -3.10\times10^6\ N/C$

Remember: it is perfectly normal for electric fields to compute to very large values.

Put the components back together:



$$E_{net} = \sqrt{E_{net\ x}^2 + E_{net\ y}^2}$$

$$E_{net} = \sqrt{\left(1.40 \times 10^6\right)^2 + \left(3.10 \times 10^6\right)^2}$$

$$E_{net} \approx 3.40 \times 10^6 \ N/C$$

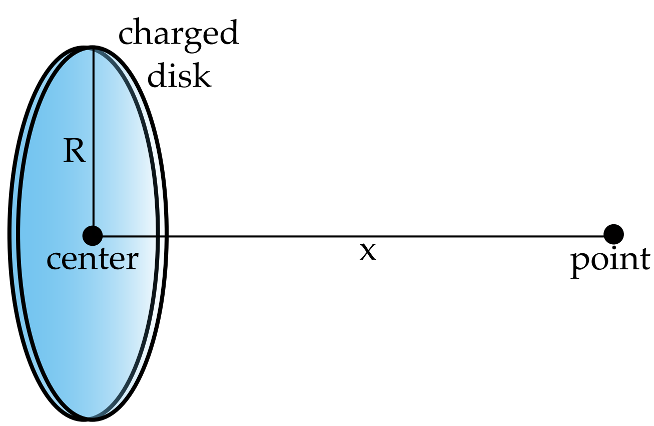
$$\theta_{net} = \tan^{-1}\left(\frac{E_{net\ y}}{E_{net\ x}}\right)$$

$$\theta_{net} = \tan^{-1}\left(\frac{3.10 \times 10^6}{1.40 \times 10^6}\right)$$

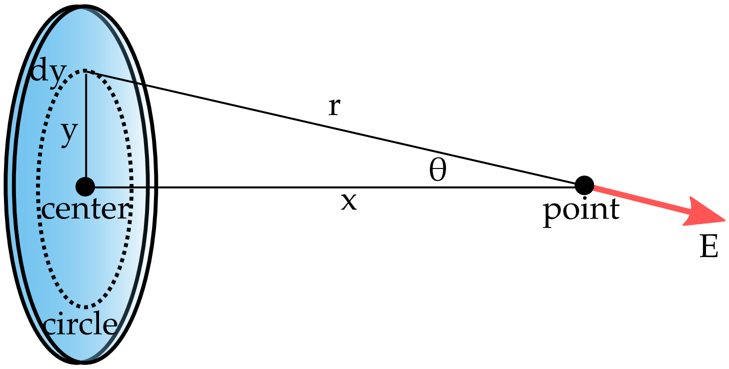
$$\theta_{net} \approx 65.7^{\circ}$$

Note: $\theta_{net} \approx 294.3^{\circ}$ as a conventional angle.

Problem 5: What is the electric field at a point at a distance x, perpendicular to the center of a disk with radius R and a uniform surface charge density σ, in terms of x, R and σ? Greek lower-case letter, sigma, is charge per area: $\sigma = \dfrac{q}{A}$ (PHY 1400 only)



Solution: a disk can be thought of as an infinite series of rings (circles) with increasing radius. The strategy is to begin with a circle of charge of radius y, and infinitesimal width dy, and take an infinite summation (integrate) from 0 to R.



$$E_{point} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}$$

$$\therefore \quad dE_{infinitesimal} = \frac{1}{4\pi\varepsilon_0}\,\frac{dq}{r^2}$$

$$dE_{infinitesimal} = \frac{1}{4\pi\varepsilon_0}\,\frac{dq}{x^2+y^2}$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\,\frac{dq}{x^2+y^2}\,\cos\theta$$

$$(\,dE_y = 0 \text{ because of symmetry})$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\,\frac{dq}{x^2+y^2}\,\frac{x}{\sqrt{x^2+y^2}}$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\,\frac{x\;dq}{\left(x^2+y^2\right)^{3/2}}$$

$$\therefore \quad E = \int dE_x = \frac{1}{4\pi\varepsilon_0}\int \frac{x\;dq}{\left(x^2+y^2\right)^{3/2}}$$

$$\text{since} \quad \sigma = \frac{q}{A} = \frac{dq}{dA} \quad \text{and} \quad l = 2\pi y$$

$$\text{then} \quad dA = 2\pi y\;dy \quad \text{and} \quad \sigma = \frac{dq}{2\pi y\;dy}$$

$$\text{or} \quad dq = 2\pi y\sigma\;dy$$

$$\text{since} \quad E = \frac{1}{4\pi\varepsilon_0} \int \frac{x\ dq}{\left(x^2+y^2\right)^{3/2}}$$

$$\therefore \quad E_{disk} = \frac{1}{4\pi\varepsilon_0} \int_0^R \frac{x\,2\pi y\sigma\ dy}{\left(x^2+y^2\right)^{3/2}}$$

$$\text{or} \quad E_{disk} = \frac{x\sigma}{4\varepsilon_0} \int_0^R \frac{2y\ dy}{\left(x^2+y^2\right)^{3/2}}$$

$$\text{let} \quad u = x^2 + y^2$$

$$\text{then} \quad du = 2y\ dy$$

$$\text{and} \quad \int \frac{2y\ dy}{\left(x^2+y^2\right)^{3/2}} = \int \frac{du}{u^{3/2}} = \int u^{-3/2}\ du = -2u^{-1/2} = \frac{-2}{u^{1/2}}$$

$$\therefore \quad \int \frac{2y\ dy}{\left(x^2+y^2\right)^{3/2}} = \frac{-2}{\sqrt{x^2+y^2}}$$

$$\text{then} \quad E_{disk} = \frac{x\sigma}{4\varepsilon_0}\left(\frac{-2}{\sqrt{x^2+y^2}}\right)_{y=0}^{y=R} = \frac{x\sigma}{2\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+y^2}}\right)_{y=0}^{y=R}$$

$$= \frac{x\sigma}{2\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+R^2}} - \left(\frac{-1}{\sqrt{x^2+0^2}}\right)\right)$$

$$= \frac{x\sigma}{2\varepsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

$$\therefore \quad E_{disk} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

QED

Part 3: Electric potential and potential energy

An important idea you should have recognized in PHY 1100/1300 was that there are almost always two different Math ways to solve Physics problems. Newton's Second Law force and acceleration calculations are vector calculations; while work-energy theorem calculations are scalar calculations, but they are Physically equivalent – they give the same answers. Usually, work-energy scalar calculations are easier.

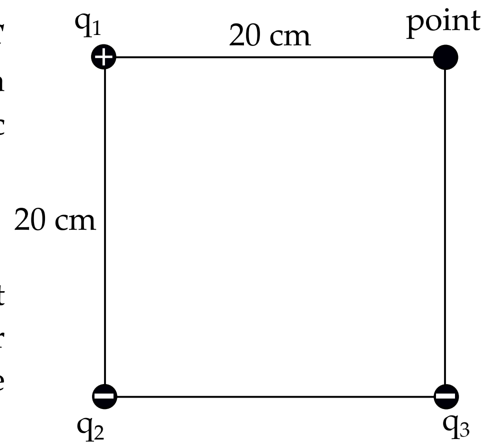
This continues in PHY 1200/1400. Electric potential and potential energy are the scalar equivalents of electric field and force vectors. For point charges:

	electric charge (quantity of matter)	
	q	
SCALAR		VECTOR
electric potential	field-potential relationship	electric field
$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$	$V = Ed$ or $V = -\int E(r)\,dr$	$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}$
energy-potential relationship		force-field relationship
$W = \Delta U_E = q\,\Delta V$		$\vec{F}_E = q\,\vec{E}$
electric potential energy	force-energy relationship (and work-energy theorem)	electric force
$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1\,q_2}{r}$	$W = Fd$ or $W = \int F(r)\,dr$	$\vec{F}_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1\,q_2}{r^2}\,\hat{r}$
	$\sum W_{nc} = \Delta K + \Delta U$	

Review work and energy from PHY 1100/1300, if you don't remember. Electric potential energy (U_E) is still a "stored" energy – energy that can be used to do work. Electric potential (V) is potential energy per unit charge. An electric potential difference ($\Delta V_{a\to b} = V_b - V_a$) is often called voltage. Note, since work and energy are scalars, a positive charge is computed as a positive number and a negative charge is computed as a negative number.

Part 3: Electric potential and potential energy Problems

Problem 1: Three charges $q_1 = +8\ \mu C$, $q_2 = -5\ \mu C$ and $q_3 = -12\ \mu C$ are fixed at three corners of a square, 20 cm on each side. What is the net electric potential at a point on the other corner?

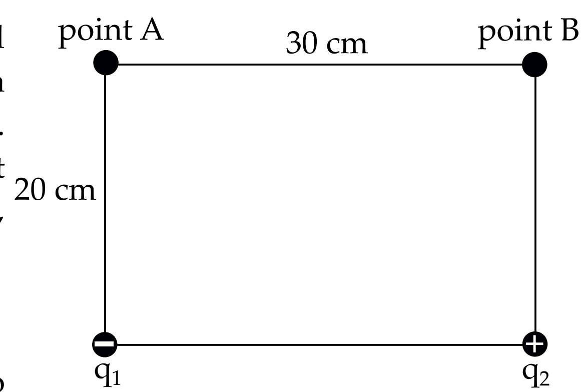


Solution 1: since electric potential is a scalar, the net potential is a simple algebraic sum. (No vector addition.) Remember, q_2 is across the diagonal of the square to the point, so $r_2 \approx 28.3\ cm$:

$$
\begin{aligned}
V_{net} &= V_1 + V_2 + V_3 \\
&= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3} \\
&= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right) \\
&\approx 9 \times 10^9 \left(\frac{8 \times 10^{-6}}{0.20} + \frac{-5 \times 10^{-6}}{0.283} + \frac{-12 \times 10^{-6}}{0.20}\right) \\
&\approx 9 \times 10^9 \left(4 \times 10^{-5} - 1.77 \times 10^{-5} - 6 \times 10^{-5}\right) \\
&\approx 9 \times 10^9 \left(-3.77 \times 10^{-5}\right) \\
V_{net} &\approx -3.39 \times 10^5\ Volts
\end{aligned}
$$

Remember: it is perfectly normal for electric potentials to compute to very large values.

Problem 2: Two point charges $q_1 = -5\ \mu C$ and $q_3 = +18\ \mu C$ are fixed to corners of a rectangle. The other corners are points A and B. How much work is required to move a test charge $Q_{test} = +2\ \mu C$ from point A to point B, in the diagram:



Solution 2: since there are two charges and two points, there are four individual potentials. We

need to find the net potentials at both points, so we can find the potential difference. Notice, the diagonal of the rectangle is need for some distances, with $r \approx 36.1\ cm$:

$$
\begin{aligned}
V_{A\ net} &= V_{A1} + V_{A2} \\
&= \frac{1}{4\pi\varepsilon_0}\ \frac{q_1}{r_{A1}} + \frac{1}{4\pi\varepsilon_0}\ \frac{q_2}{r_{A2}} \\
&= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{A1}} + \frac{q_2}{r_{A2}}\right) \\
&\approx 9\times 10^9\left(\frac{-5\times 10^{-6}}{0.20} + \frac{18\times 10^{-6}}{0.361}\right) \\
&\approx 9\times 10^9\left(-2.5\times 10^{-5} + 5.0\times 10^{-5}\right) \\
&\approx 9\times 10^9\left(2.5\times 10^{-5}\right) \\
V_{A\ net} &\approx 2.25\times 10^5\ Volts
\end{aligned}
$$

$$
\begin{aligned}
V_{B\ net} &= V_{B1} + V_{B2} \\
&= \frac{1}{4\pi\varepsilon_0}\ \frac{q_1}{r_{B1}} + \frac{1}{4\pi\varepsilon_0}\ \frac{q_2}{r_{B2}} \\
&= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{B1}} + \frac{q_2}{r_{B2}}\right) \\
&\approx 9\times 10^9\left(\frac{-5\times 10^{-6}}{0.361} + \frac{18\times 10^{-6}}{0.20}\right) \\
&\approx 9\times 10^9\left(-1.385\times 10^{-5} + 9.0\times 10^{-5}\right) \\
&\approx 9\times 10^9\left(7.62\times 10^{-5}\right) \\
V_{B\ net} &\approx 6.85\times 10^5\ Volts
\end{aligned}
$$

$$
\begin{aligned}
W &= q\ \Delta V \\
W_{AB} &= Q_{test}\ \Delta V_{AB} \\
W_{AB} &= Q_{test}\left(V_B - V_A\right) \\
&= +2\times 10^{-6}\left(6.85\times 10^5 - 2.25\times 10^5\right) \\
W_{AB} &\approx 0.92\ J
\end{aligned}
$$

Notice, work and energy is still measured in Joules. Also notice, the work is small, because the charges are small.

Part 4: Electric current, capacitance and capacitor circuits

When we were learning about electric forces and fields, the charges were not moving. We were covering electrostatics. (Yes, Newton's Second Law says that any force can cause acceleration, but we didn't explore that acceleration.) We will now begin studying moving charges – electrodynamics.

---

electric current

An electric current is a directed flow of charged particles, expected to be through a conductor (a metal wire). The analogy is with a flow of hot water through a pipe.

$$I = \frac{\Delta q}{\Delta t}$$

Since the current is "pumped" by an electric potential difference, the current carries electric potential energy with it.

Note: electric circuits were invented a century before the proton and electron were discovered. The current was originally assumed to be a flow of positive charges. We now know that a real current is a flow of negative electrons in the opposite direction. For simplicity, we will continue to use the positive CONVENTIONAL CURRENT, not the real negative current for the simple circuits in PHY 1200/1400. This is not OK for more complex circuits. The real charge flow matters for modern "solid state" circuits. This is one reason why you can't install a battery backwards in modern gizmos.

---

definitions of capacitance

A capacitor (or condenser) is a device that stores electrical energy as an electric field. A capacitor is any two conducting materials separated by an insulating material. Equal, but opposite electric charge can be built up on the opposite conductors.

Since like charges attract, an electric potential difference (a voltage) must do work to move electric charge from one side of the capacitor to the other, building up an electric field between the two sides. Therefore, the work transfers electric potential energy to the capacitor. Capacitance is defined:

$$C = \frac{q}{V}$$

The unit of capacitance is the Farad, F. Since one Coulomb of charge is an enormous amount of charge, one Farad of capacitance is also an enormous amount of capacitance. You will see microFarad, $\mu F = 10^{-6}\ F$; nanoFarad, $nF = 10^{-9}\ F$; and even picoFarad, $pF = 10^{-12}\ F$ in realistic problems.

A capacitor is real device, that you can build yourself. All you need are two conducting sheets (sheets of aluminum foil are fine), sandwiching an insulating sheet (a sheet of moist paper works well). For a parallel-plate capacitor:

$$C = \kappa\, \varepsilon_0\, \frac{A}{d}$$

where: A is the area of the conducting sheets (in square meters), d is the distance between the conducting sheets (in meters, of course), the Greek lower-case letter, kappa, $\boldsymbol{\kappa}$ is the "dielectric constant" for the efficiency of the insulation, and ε_0 is still the electric constant. Note: manufactured capacitors usually roll up the plates into a cylinder shape.

The electric field in a parallel-plate capacitor is (without proof):

$$E = \frac{V}{d} \qquad \text{or} \qquad E = \frac{q}{\kappa \varepsilon_0 A}$$

Since "σ", the Greek lower case letter "sigma", is the surface charge density (charge per area) $\sigma = \frac{q}{A}$, we can rewrite:

$$E = \frac{\sigma}{\kappa \varepsilon_0}$$

The energy stored in a capacitor is (without proof):

$$U = \tfrac{1}{2}\, qV = \tfrac{1}{2}\, CV^2 = \tfrac{1}{2}\, \frac{q^2}{C}$$

If you took CHM 1200, you will remember that a galvanic cell (a battery) produces electrical energy by a chemical reaction – it stores energy in chemical bonds. If batteries and capacitors both store energy, why do we need both? A battery stores a lot of energy, but produces it slowly – it has low power, but can last hours. A capacitor stores a small amount of energy, but releases it very quickly – it has very high power, but discharges in a split second.

Since charges cannot instantly enter or exit a capacitor.

---

series and parallel capacitor circuits

A capacitor circuit is any device that transfers electric potential energy from a source into a capacitor. The energy source is a battery (the potential source). The battery and capacitor are connected by conductors (metal wire). The symbol for a capacitor is a double line of equal length. The symbol for a battery is a double line – one long for the positive electrode (the cathode), one short for the negative electrode (the anode). They are connected are single lines for the wires.

If there is more than one capacitor, and you have enough wire, you can connect the capacitors in many different ways.

The two basic circuit connections are series (the circuit elements are wired one after another, so that the charges can only distribute along one path) and parallel (the circuit elements are wired across junction points, "intersections" in the wiring, so the charges can distribute along multiple paths). Note, the rules for drawing circuit diagrams are loose. It is possible to draw very different looking diagrams for equivalent circuits. You are responsible for tracing the wiring and recognizing series versus parallel.

For a series capacitor circuit, the equivalent capacitance adds by reciprocal:

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

In addition, the charge is constant for all elements of a series capacitor circuit, but the voltage sums up: $V_{series} = V_1 + V_2 + V_3 + \dots$

For a parallel capacitor circuit, the equivalent capacitance is a simple sum:

$$C_{parallel} = C_1 + C_2 + C_3 + \dots$$

In addition, the voltage is constant for all elements of a series capacitor circuit, but the charge sums up: $q_{parallel} = q_1 + q_2 + q_3 + \dots$

Part 5: Ohm's Law and resistor circuits

Remember: since an electric current is "pumped" by an electric potential, the current transfers electric potential energy.

A resistor is anything that does work with electrical energy. A flashlight uses electrical energy to do the work of producing light. An air conditioner uses electrical energy to do the work of cooling air. A TV uses electrical energy to do the work of converting an electrical signal into picture and sound. Remember, we are using the Physics definition of work – work transfers or transforms energy.

The unit of resistance is the Ohm, abbreviated "Ω", the Greek capital letter "omega." Most real devices do not indicate their resistance values. The major exception is sound/audio electronics, which is often labeled by their impedance, which is equivalent to resistance.

A resistor circuit is any device that transfers electric potential energy from a source to do work. The energy source is a battery (the potential source); the work is done by a resistor; they are connected by conductors (metal wire). The symbol for a resistor is a zig-zag line.

Resistor circuits obey Ohm's Law:

$$V = IR$$

Multiple resistors can also be connected in series (one after another) or parallel (across junction points). Series and parallel is a property of the wiring, not what's plugged into the circuit.

For a series resistor circuit, the equivalent resistor is a straightforward sum:

$$R_{series} = R_1 + R_2 + R_3 + \ldots$$

In addition, the current is constant for all elements of a series capacitor circuit, but the voltage adds up: $V_{series} = V_1 + V_2 + V_3 + \ldots$

For a parallel resistor circuit, the equivalent resistor adds by reciprocal:

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

In addition, the voltage is constant for all elements of a series capacitor circuit, but the charge sums up: $I_{parallel} = I_1 + I_2 + I_3 + \ldots$

By the way, rheostats and potentiometers are adjustable resistors.

---

resistivity and internal resistance

Real metals are not perfect conductors. Their resistance is low, but not zero. Some electric energy is lost as heat in real metal wires. (This is how electrical fires can start. If the current is too high, the wire gets hot enough that nearby materials can catch fire.) The conductive efficiency of metals can be measured by their resistivity, abbreviated "**ρ**". Notice: this is the Greek letter, lower case "rho", which was used for mass density in PHY 1100/1300. You're supposed to tell the difference by context.

The resistance of a real wire is:

$$R = \rho \frac{L}{A}$$

Real batteries are not perfect potential sources. They have an internal resistance which lowers the voltage produced by the battery. Some of the voltage is lost getting out of a real battery. Internal resistance is abbreviated "r".

Kirchhoff's Rules

Many circuits are too complex to handle as series or parallel. They are sometimes called mesh or network circuits. In this case, use Kirchhoff's Rules instead. There are two rules.

Kirchhoff's Loop Rule (also called the voltage rule) – for any complete loop (starting at any point in the circuit, follow the wiring and trace your way around the circuit, and end back at the original point) – is:

$$\sum V = \sum IR$$

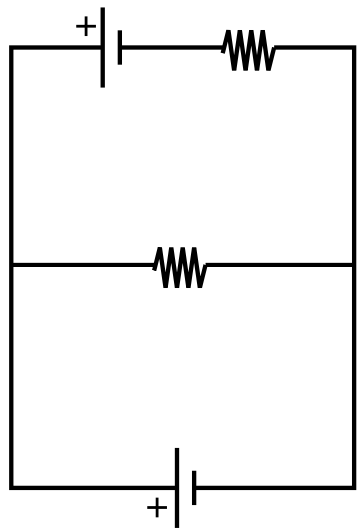
Notice: this is a modified version of Ohm's Law. It is also a restatement of conservation of energy: for any complete loop, the total energy added by the potentials must equal the total work done by the resistors.

Kirchhoff's Junction Rule: (also called the current rule) – at any junction point (any intersection of multiple wires) – is:

$$\sum I_{in} = \sum I_{out}$$

Notice: this is a restatement of conservation of charge: at any junction point, the total current entering the junction must equal the total current exiting the junction.

Kirchhoff's Rules can be applied to series and parallel circuits, but that is considered unnecessary. We will use Kirchhoff's Rules for "three-branched" circuits, which look something like this:



Three-branched circuits have three branches joining the two junction points. They also have three loops, which I call the top loop, the bottom loop and the outside loop.

Each branch has a current, which are often the unknowns. The usual procedure applies the loop rule to two of the three loops, and the junction rule once. Notice, you will have a system of three equations with three unknowns. You can use any Math technique you want. The basic one is a double substitution. If you want to impress your professor, use determinants or matrices.

Part 6: Magnetic forces and fields

Electricity and magnetism are closely related. Anything that can be done with electricity can also be done with magnetism. We are going to repeat a lot of the things we did before with electricity, with magnetism.

For example: do you see the similarity between the magnetic force/field and the electric force/field equations?

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad \text{related to} \qquad \vec{F}_E = q\ \vec{E}$$

Notice: there are two major differences. First, magnetic force requires the electric charge be moving, while electric force does not. There is both electrostatics and electrodynamics, but there is no magnetostatics, only magnetodynamics. Second, the magnetic force equation is a three-vector equation, while the electric force equation is a two-vector equation. Magnetic effects are three-dimensional, but electric effects are two-dimensional.

The fact that there is electrostatics, but not magnetostatics has deep Physical meaning – that ultimately leads to Einstein's relativity theories. We are not covering that. It's problem solving, we're worried about. That's supposed to be straightforward — include the necessary velocity (or current) in your calculations.

Unfortunately, the fact that magnetic effects are three-dimensional makes calculations messy. Strictly speaking, $\vec{v} \times \vec{B}$ is a vector cross product (PHY 1400 only)

So we give you a shortcut:

magnitude: $F_B = q\,vB \sin \phi_{vB}$

direction: right-hand-rule, RHR:

> Hold your right hand with fingers straight and thumb out (do not flex your fingers or thumb);
> step 1: point your fingers in the direction of charge velocity vector,
> step 2: point your thumb in the direction of magnetic field vector,
> step 3: your palm faces the the direction of magnetic force vector.
> Note: there are alternate ways to set up the RHR.

Part 7: Electromagnetic induction

Electricity and magnetism are so closely related that it is possible to convert one to the other. In more advanced Physics, they are actually considered opposite sides of the same coin, called electromagnetism.

Faraday's Law gives you the induced electromotive force (emf – the voltage) caused by a changing magnetic flux:

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$$

Part 8: AC circuits

When we were covering resistor circuits before, we assumed DC electricity. Direct Current means a constant electric current. Batteries give DC electricity.

LC -circuits, tank circuit

AC electricity means an oscillating current. Electrical outlets give AC electricity. Instead of a steady flow of electrons, the electrons are vibrating. Instead of electrons directly carrying energy from the source through the circuit, vibrating electrons set up a wave that carries energy around the circuit.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Part 9: Light and geometric optics

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electromagnetic waves

Unlike sound waves, light waves are not mechanical waves – they are not an oscillation of matter.

Light is an energy oscillation of perpendicular electric and magnetic fields – light is an electromagnetic wave:

$$E = E_0 \sin(kx - \omega t) \qquad \text{and} \qquad B = B_0 \sin(kx - \omega t)$$

that travel at a constant speed that is the ratio of electric and magnetic fields:

$$c = \frac{E}{B}$$

The fields are perpendicular to each other, and both perpendicular to the traveling speed. Notice: light waves are three-spatial dimension transverse waves – if E is along the x-axis and B is on the y-axis, c will be along the z-axis. Light waves that do not need a medium to travel through – they can radiate through a vacuum.

The exact orientation of the fields compared to the traveling direction is called the polarization.

Since $v = \dfrac{\omega}{k}$, the constant speed of light in a vacuum:

$c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}792{,}458\ m/s.$

Note: the speed of light in a medium is always less than in a vacuum.

The Poynting vector gives the light (power) intensity. It is:

$S = \dfrac{1}{\mu_0} EB$

---

the electromagnetic spectrum

There are different types of light because they have different frequencies and wavelengths.

From longest wavelength/lowest frequency to shortest wavelength/highest frequency, they are: radio/microwave/infrared/visible/ultraviolet/x-rays/gamma rays.

Visible light means light that humans can see (with their eyes). Visible light ranges between about 400 nm to 700 nm wavelength. Visible light is further separated into the traditional seven major colors of the rainbow – red/orange/yellow/green/blue/indigo/violet – often abbreviated ROY-G-BIV.

The other spectrum bands require special equipment if humans are to perceive them. There are some animals that can sense light that humans cannot.

Strictly speaking, light is most properly described by quantum theory, but that is gross overkill in most real applications.

---

reflection

Light has some interesting interactions with matter. Simple geometry can illustrate much of light's behavior. Be careful: the way our brains perceive light hides many aspects of what light is – there are many properties of light that we cannot see.

Whenever light hits a surface between different media (materials), some of the light will bounce off. (We ignore the light that is absorbed or passes through.) The incident angle will equal the reflection angle (measured to a normal [perpendicular] line to the surface).

If the surface is planar (flat) and very smooth (like polished metal), parallel incident rays remain parallel after reflection. A flat mirror will produce an image that duplicates the original object, but appears to be behind the mirror.

If the surface is spherical (curved like part of a sphere), it reflects light in specific directions – spherical mirrors focus light.

Note: we always draw 3-D spherical surfaces as 2-D circular arcs, for simplicity's sake.

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refraction

Whenever light travels from one medium (material) into another, it will change direction, because its speed changes. (We ignore the light that is absorbed or reflected.)

The speed change is measured by the index of refraction:

$$n = \frac{c}{v}$$

The index of refraction depends on the material. Water has an index of refraction of 1.33, most optical plastics of 1.5 to 1.6, most optical glasses of 1.5 to 1.8. You are not expected to memorize any values.

The direction change is measured by Snell's Law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Be careful, angles are measured to a line perpendicular (a normal line) to the surface, not to the surface.

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lenses

A lens is a piece of glass (or plastic) curved in a particular way so that it refracts light in specific directions – lenses focus light.

A ray trace diagram is a simple way to show how lenses focus light and form images. The diagram has a horizontal line called the optical axis, pointing to the right. This axis reminds you that the light is traveling from left to right. Distances are measured along the axis.

A converging, or convex, lens (thicker in the center than at the rim) is represented by a vertical line with outward pointing arrowheads, centered on the axis. A diverging, or concave, lens (thinner in the center than at the rim) is drawn as a vertical line with inward pointing arrowheads. (This is not an x-y axis.) Dots are added to the axis to represent the focal point and the symmetric reciprocal focal point. Notice: we use the "thin-lens approximation" where lens thickness is ignored. (In other words, this technique isn't used by Nikon, Canon, Sony, etc.)

There are a near infinite number of rays that can be traced. Only two are required to locate an image. We want you to trace three rays.

for a converging lens:

ray #1: draw a straight line from the top of object, parallel to optical axis until it reaches the lens plane, then refract straight through the focal point, and continue straight.

ray #2: draw a straight line from the top of object, straight through lens vertex, and continuing straight (does not refract).

ray #3: draw straight line from top of object, through the reciprocal focal point until it reaches lens plane, then refract parallel to optical axis.

*ray #3: if object distance is less than focal length: draw straight line from reciprocal focal point to the top of object, continuing straight until it reaches lens plane; then refract parallel to optical axis, both backtracing and continuing forward. You will need to backtrace ray #1 and #2 now.

for a diverging lens:

ray #1: draw a straight line from the top of object, parallel to optical axis until it reaches lens plane, then refract with a straight backtrace from the focal point, that continues straight past the lens.

ray #2: draw a straight line from the top of object, straight through lens vertex, continuing straight through the focal point (does not refract).

ray #3: draw a straight line connecting the top of object with the reciprocal focal point; where this line crosses the lens plane, refract by drawing a straight line parallel to optical axis, both backtracing and continuing forward.

Since the three rays begin at the top of object, where they intersect again is the top of the image. You are not expected to be a great artist, so your ray trace does not have to be perfect, but it does need to be reasonable. For example, the intersection does not have to be exact, but it should be close. Spend a dollar and buy a ruler, so that you can draw straight lines. Make the ray trace neat, to reasonable scale and fairly large, so that you can write on and read off your information.

There are also lens image formulae, of course:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$mag = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Your ray trace and calculations should match. Note, most lenses are small, with focal lengths measured in millimeters. It is often quicker to convert all distances to millimeters, instead of meters like in all other Physics topics.

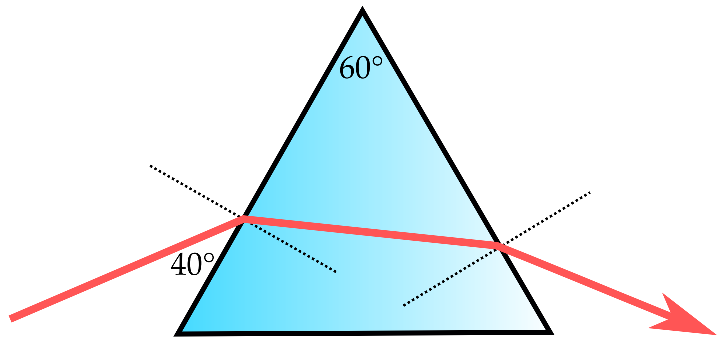
Remember your sign conventions:

	in front (left of lens)	**behind (right of lens)**
f (focal point)	negative (diverging)	positive (converging)
d_o **(object)**	positive	negative
d_i **(image)**	negative (virtual)	positive (real)

diffraction

Part 9: Light and geometric optics Problems

Problem: A laser is shined into a 60° triangular prism. If the beam hits at 40° to the glass surface, at what angle will the beam emerge on the other side? The glass has an index of refraction $n = 1.69$.



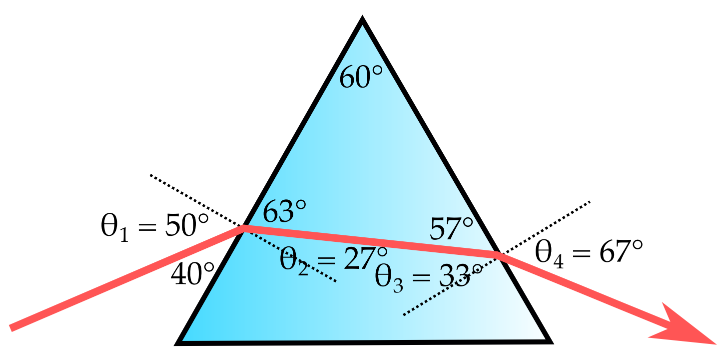
Solution: this is a two part problem: air-to-glass refraction as the beam enters the prism, and glass-to-air as the beam exits the prism.

Remember, angles are measured to a line perpendicular to the surface (not to the surface). Therefore $\theta_1 = 50^{\circ}$.

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ \therefore \quad \sin\theta_2 &= \frac{n_1 \sin\theta_1}{n_2} \\ \sin\theta_2 &= \frac{1.00 \sin 50^{\circ}}{1.69} \\ \sin\theta_2 &\approx 0.453 \\ \theta_2 &\approx \sin^{-1} 0.453 \\ \theta_2 &\approx 27.0^{\circ} \end{aligned}$$

From basic geometry, the beam reaches the other side of the prism at $\theta_3 = 33^{\circ}$.

$$\begin{aligned} n_3 \sin\theta_3 &= n_4 \sin\theta_4 \\ \therefore \quad \sin\theta_4 &= \frac{n_3 \sin\theta_3}{n_4} \\ \sin\theta_4 &\approx \frac{1.69 \sin 33.0^{\circ}}{1.00} \\ \sin\theta_4 &\approx 0.920 \\ \theta_4 &\approx \sin^{-1} 0.920 \\ \theta_4 &\approx 67.0^{\circ} \end{aligned}$$



Formulae:

$v = f\lambda$ or $v = \lambda\,\nu$

$T = \dfrac{1}{f}$ or $f = \dfrac{1}{T}$

$v = \dfrac{\lambda}{T}$

$k = \dfrac{2\pi}{\lambda}$ $\omega = 2\pi f$

$y(x,t) = A\cos(kx - \omega t)$

$v(x,t) = \dfrac{\partial\ y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$

$a(x,t) = \dfrac{\partial\ v(x,t)}{\partial t} = -\,\omega^2 A\cos(kx - \omega t) = -\,\omega^2\ y(x,t)$

$v_{string} = \sqrt{\dfrac{F_T}{\mu}} \equiv \sqrt{\dfrac{F_T}{{}^{m}/_{l}}}$

$v_{sound} \approx 331\ m/s + 0.60\ T\,(°C)$

$I = \dfrac{P}{A}$

$\beta = -\,10\log\dfrac{I}{I_0}$

$f_{observer} = f_{source}\left(\dfrac{v_{sound}\ \pm\ V_{observer}}{v_{sound}\ \mp\ V_{source}}\right)$

$f_n = n\dfrac{v}{2L}$ and $\lambda_n = \dfrac{2L}{n}$ where n = 1, 2, 3, …

$f_n = n\dfrac{v}{4L}$ and $\lambda_n = \dfrac{4L}{n}$ where n = 1, 3, 5, … (odd numbers only)

$F_E = \dfrac{1}{4\pi\varepsilon_0}\ \dfrac{q_1\ q_2}{r^2}$

$E = \dfrac{1}{4\pi\varepsilon_0}\ \dfrac{q}{r^2}$

$\vec{F}_E = q\ \vec{E}$

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

$$U_E = \frac{1}{4\pi\varepsilon_0} \frac{q_1 \, q_2}{r}$$

$$W = \Delta U_E = q \ \Delta V$$

$$I = \frac{\Delta q}{\Delta t}$$

$$C = \frac{q}{V} = \kappa \, \varepsilon_0 \, \frac{A}{d}$$

$$E = \frac{V}{d}$$

$$\sigma = \frac{q}{A}$$

$$U = \tfrac{1}{2} \, qV = \tfrac{1}{2} \, CV^2 = \tfrac{1}{2} \, \frac{q^2}{C}$$

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

$$C_{parallel} = C_1 + C_2 + C_3 + \ldots$$

$$V = IR$$

$$R_{series} = R_1 + R_2 + R_3 + \ldots$$

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$$R = \rho \frac{L}{A}$$

$$\sum V = \sum IR$$

$$\sum I_{in} = \sum I_{out}$$

Greek alphabet:

letter	capital	lower case	English sound
alpha	A	α	a
beta	B	β	b
gamma	Γ	γ	g
delta	Δ	δ	d
epsilon	E	ϵ or ε	e
zeta	Z	ζ	z
eta	H	η	ē
theta	Θ	θ	th
iota	I	ι	i
kappa	K	κ	k
lambda	Λ	λ	l
mu	M	μ	m
nu	N	ν	n
xi	Ξ	ξ	x
omicron	O	o	o
pi	Π	π	p
rho	P	ρ	r
sigma	Σ	σ	s
tau	T	τ	t
upsilon	Υ	υ	u
phi	Φ	ϕ or φ	ph
chi	X	χ	kh
psi	Ψ	ψ	ps
omega	Ω	ω	ō